

Note on Noncommutative Tachyon in Matrix Models

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The solution representing a brane-anti-brane system in matrix models breaks the usual matrix spacetime symmetry. We show that the spacetime symmetry on the branes is not breaking, rather appears as a combination of the matrix spacetime transformation and a gauge transformation. As a result, the tachyon field, itself an off-diagonal entry in longitudinal matrices, transforms nontrivially under rotations, decomposing into tensors of different ranks. We also show that the tachyon field can never be gauged away, and conjecture that this field is related to the usual complex scalar tachyon by a field redefinition. We also briefly discuss tachyon condensation.

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1. Introduction

Tachyon condensation has been one of the focus points of recent research in string theory [1,2]. One apparent motivation in studying this issue is to understand the dynamic process of the brane-anti-brane annihilation. More profoundly, revelation of elements in this problem will shed new lights on the deep connection between the open string sector and the closed string sector, and on the old problem of background independence in string theory. The tachyon condensation problem has been proven considerably simplified when a large B field is present [3-6], thanks to some new features of noncommutative field theory [8].

As conjectured recently by Witten [7], it may be possible to realize any configuration in, for instance the IIB theory, by starting with infinitely many D9 and anti-D9 branes. It is also very interesting to ask whether it is possible to “derive” the IIB matrix model or some revision of it [9,10], by starting with D9-anti-D9 system and turning on tachyon condensation. We believe that the answer to this question is likely yes. The reason for this belief is the following. Consider for instance a single D9-brane with a large B field switched on. The open string sector decouples from the closed string sector in the Seiberg-Witten scaling limit [8]. If the rank of B is maximal (in the Euclidean spacetime), then one can rewrite the noncommutative Yang-Mills theory as a 0-dimensional matrix model in an operator formulation. Since there is no tachyon in this system, it is not possible to realize other D-branes and closed strings yet. By adding anti-D9-branes, one would get a matrix model different from the IIB matrix model: If both D9-branes and anti-D9-branes are immersed in the same B field background, again one has a noncommutative field theory, one that is different from the IIB matrix model in an operator formulation.

One can study this problem by reversing the above procedure. One starts with the IIB matrix model and construct D9-anti-D9 solution. This solution is a system quite different from D9-anti-D9 embedded in the same constant B field background. The reason for this is quite simple. If the D9-brane is represented by a solution $[X^i, X^j] = i\theta^{ij}$, then an anti-D9-brane is represented by a solution $[X^i, X^j] = -i\theta^{ij}$, corresponding to reversing the orientation of the D9-brane solution by making reflection of 5 dimensions. However, a D-brane solution in matrix models does not come for free, it always carries a background gauge field F_{ij} which is just the inverse of θ^{ij} . Indeed for this solution, as first observed in [11], a noncommutative super Yang-Mills theory directly results from the matrix model. For the brane-anti-brane solution, there are different background gauge

fields on D9-branes and anti-D9-branes. It is not known how to write down a simple field theory with an associative algebra generalizing the star product. On the D9-branes, the star product is defined with the noncommutative parameters θ^{ij} , while on the anti-D9-branes, the star product is defined with $-\theta^{ij}$. If one naively generalizes the product to include both, one will not get an associative algebra.

As we mentioned before, if one starts with a background in which the gauge field strengths on all branes are the same, then one has the usual noncommutative field theory. Other different field strengths can be achieved by adding perturbations. To simplify the situation, we shall focus on two coordinates, and the resulting theory can either be regarded as D-string anti-D-string in the IIB matrix model, or D2-brane anti-D2-brane in matrix theory. The matrix model solution of this configuration explicitly realizes the above idea. As a result, the solution breaks the usual matrix model rotational symmetry. This can be seen in two different ways. The simpler one is to simply take a look at the solution $X^1 = \text{diag}(x^1, x^1)$, $X^2 = \text{diag}(x^2, -x^2)$ with $[x^1, x^2] = i\theta$. The first entries represent the D-string, and the second entries represent the anti-D-string. This solution is not invariant under the rotation $\delta X^i = \epsilon \epsilon^{ij} X^j$. The second way to see this is to write $X^2 = \text{diag}(x^2, x^2) + \text{diag}(0, -2x^2)$. The second term can be regarded as a perturbation in the gauge field on the second string and this field depends explicitly on coordinates. This is a puzzle, since we believe that even though the field strengths on different strings are different, the whole system is invariant under a rotation. We will see in the next section that the rotational symmetry is still present in the solution. This symmetry is a mixture of the matrix rotation and a gauge transformation. In sect.3, we shall see that with the presence of gauge fields, the mixture is more complicated. This is not surprising, since gauge fields can be understood as longitudinal oscillations of branes, thus it is intimately related to spacetime symmetry.

The scalar nature of the tachyon presents another puzzle. We expect this mode to arise from the off-diagonal entries of X^i , and these entries carry longitudinal spacetime indices. This puzzle again is related to the lacking of matrix rotational symmetry. We will in sect.4 see that one of the complex off-diagonal entry can be gauged away, the remaining off-diagonal entry is tachyonic. Under the rotation about a point, the tachyon can be divided into modes with different tensor structures. We attribute this unusual feature to the bad representation of brane-anti-brane system in matrix models. We believe that if one can write down an associative algebra without breaking the matrix rotational symmetry, the scalar nature of the tachyon will become apparent. We briefly discuss tachyon condensation in the final section.

A paper discussing related issues appeared very recently [14].

2. Rotational Invariance

In the Euclidean signature, the brane-anti-brane solution can be represented as

$$X^1 = \text{diag}(x^1, x^1), \quad X^2 = \text{diag}(x^2, -x^2). \quad (2.1)$$

Here x^1, x^2 are noncommutative coordinates satisfying

$$[x^1, x^2] = i\theta. \quad (2.2)$$

Apparently, the brane-anti-brane system still respects the rotational invariance on the (x^1, x^2) plane, this is a consequence of the first quantized strings for them the world-sheet action is invariant under rotations. For an infinitesimal rotation, $\delta x^1 = -\epsilon x^2, \delta x^2 = \epsilon x^1$, we expect the matrices X^1, X^2 transform in the same way. This is not the case for the ansatz (2.1), since X^1 is proportional to the identity matrix while X^2 is proportional to σ_3 . Thus we can hardly obtain a manifestly rotationally invariant action or Hamiltonian starting with ansatz (2.1).

In matrix models, in addition to translational and rotational symmetry, the only other symmetry readily available is gauge symmetry. Applying a rotational transformation directly to X^1, X^2 would bring us away from the ansatz (2.1). To come back to (2.1), one may try to apply a gauge transformation. It is easy to guess what kind of gauge parameter is needed. Since the first diagonal entries transform correctly under the matrix spacetime transformation, thus the gauge parameter has a vanishing first diagonal entry. Assume $g = \text{diag}(0, f)$, where f is a function of $x^{1,2}$, the gauge transformation takes the form $\delta_g X^i = i[g, X^i]$. Now apply the rotational transformation

$$\delta_\epsilon X^1 = \text{diag}(-\epsilon x^2, \epsilon x^2), \quad \delta_\epsilon X^2 = \text{diag}(\epsilon x^1, \epsilon x^1).$$

Applying further a gauge transformation, in order to keep the ansatz (2.1) invariant, we need

$$\begin{aligned} \epsilon x^2 + i[f, x^1] &= -\epsilon x^2, \\ \epsilon x^1 - i[f, x^2] &= -\epsilon x^1. \end{aligned} \quad (2.3)$$

The apparent solution to the above conditions is

$$f = -\frac{\epsilon}{\theta}[(x^1)^2 + (x^2)^2]. \quad (2.4)$$

The corrected rotational transformation therefore is

$$\begin{aligned}\delta X^1 &= (\delta_\epsilon + \delta_g)X^1 = \text{diag}(-\epsilon x^2, -\epsilon x^2), \\ \delta X^2 &= (\delta_\epsilon + \delta_g)X^2 = \text{diag}(\epsilon x^1, -\epsilon x^1).\end{aligned}\tag{2.5}$$

This is of course a symmetry since both terms on the R.H.S. are symmetries in the matrix model.

Note that the gauge transformation with parameter (2.4) is rather singular in the usual sense, for f diverges for large r^2 . This is not surprising however, since the rotational transformation itself diverges too.

Similarly, ansatz (2.1) does not conform with the conventional translational transformation $\delta X^i = a^i \times 1_2$, where 1_2 is the identity matrix. To preserve the form (2.1), the conventional transform is to be accompanied by a gauge transformation with a gauge parameter

$$g = \text{diag}(0, -2(a^2/\theta)x^1).\tag{2.6}$$

3. The Gauge Fields

The same issue of maintaining rotational transformation remains when we perturb from the constant background (2.1). For instance, consider switching on the $U(1)$ gauge field corresponding to the center of mass degree of freedom. In case of two coincident D-branes, the ansatz in matrix models is [11]

$$X^i = (x^i + \theta \epsilon^{ij} A_j) 1_2.\tag{3.1}$$

The commutator $[X^1, X^2]$ is given by

$$[X^1, X^2] = i\theta(1 + \theta F_{12}) 1_2,\tag{3.2}$$

where F_{12} is the noncommutative field strength

$$F_{12} = \partial_1 A_2 - \partial_2 A_1 - i[A_1, A_2],\tag{3.3}$$

and we implicitly used the star product defined by the noncommutative parameter θ . Thus, the action is proportional to

$$\int \frac{d^2x}{2\pi\theta} \theta^2 (1 + \theta F_{12})^2.\tag{3.4}$$

As noticed in [12], θ^2 is to be interpreted as the open string metric factor G^{ij} . The integral factor is simply the phase space factor corresponding to the matrix trace.

The correct ansatz generalizing both (2.1) and the usual brane-brane system is

$$X^1 = (x^1 + \theta A_2)1_2, \quad X^2 = (x^2 - \theta A_1)\sigma_3. \quad (3.5)$$

The commutator $[X^1, X^2]$ is the same as in (3.2) with an additional factor σ_3 . This does not change the action. Again, under the usual rotational transformation $\delta_\epsilon X^i = -\epsilon\epsilon^{ij}X^j$, the above ansatz is not preserved. We expect that with the coordinates rotation $\delta x^i = -\epsilon\epsilon^{ij}x^j$, the gauge field A_i transform in the same fashion, namely $\delta A_i = -\epsilon\epsilon_{ij}A_j$. This together with the ansatz (3.5) implies that an additional gauge transformation is again needed:

$$\delta X^i = \delta_\epsilon X^i + i[g, X^i]. \quad (3.6)$$

Again, g is diagonal and assumes the form $g = \text{diag}(0, 2\epsilon f)$. The conditions that f must meet are

$$\begin{aligned} i[f, x^1 + \theta A_2] &= -(x^1 - \theta A_1), \\ i[f, x^2 - \theta A_1] &= x^1 + \theta A_2. \end{aligned} \quad (3.7)$$

The commutators in the above can be written as covariant derivatives, so formally these conditions can be written as

$$D_i f = -i\epsilon^{ij}D_j. \quad (3.8)$$

With the presence of the other $U(1)$ gauge field B_i corresponding to the difference of gauge fields on the two branes, the correct ansatz is

$$\begin{aligned} X^1 &= (x^1 + \theta A_2)1_2 + \theta B_2\sigma_3, \\ X^2 &= (x^2 - \theta A_1)\sigma_3 - \theta B_11_2. \end{aligned} \quad (3.9)$$

To see that this is indeed the correct ansatz, we write the above formulas in the component form

$$\begin{aligned} X^1 &= \text{diag}(x^1 + \theta(A_2 + B_2), x^1 + \theta(A_1 - B_1)), \\ X^2 &= \text{diag}(x^2 - \theta(A_1 + B_1), -[x^2 - \theta(A_1 - B_1)]). \end{aligned} \quad (3.10)$$

Indeed we see that $A_i + B_i$ is the gauge field living on the brane, and $A_i - B_i$ is the gauge field living on the anti-brane. Again it is the second entries on the anti-brane that break the usual matrix rotational symmetry. To restore the coordinates rotational symmetry, we

need to form a gauge transformation with a gauge parameter of the form $g = \text{diag}(0, 2\epsilon f)$ and f satisfies

$$\begin{aligned} i[f, x^1 + \theta(A_2 - B_2)] &= -(x^2 - \theta(A_1 - B_1)), \\ i[f, x^2 - \theta(A_1 - B_1)] &= x^1 + \theta(A_2 - B_2). \end{aligned} \quad (3.11)$$

Or more compactly

$$D_i(A - B)f = -i\epsilon^{ij}D_j(A - B), \quad (3.12)$$

where $D_i(A - B)$ is the covariant derivative defined with respect to the gauge field $A - B$.

The commutator $[X^1, X^2]$ assumes a little more complicated form than for the brane-brane system

$$[X^1, X^2] = i\theta \left[(F_{12} + \theta^{-1})\sigma_3 + \tilde{F}_{12}1_2 \right], \quad (3.13)$$

where it is not surprising to see the term θ^{-1} since the B field is $-\theta^{-1}$. The two $U(1)$ field strengths are defined by

$$\begin{aligned} F_{12} &= \partial_1 A_2 - \partial_2 A_1 - i[A_1, A_2] - i[B_1, B_2], \\ \tilde{F}_{12} &= D_1(A)B_2 - D_2(A)B_1, \end{aligned} \quad (3.14)$$

where the covariant derivatives $D_i(A)$ are defined against the gauge field A_i . It is easy to understand why these covariant derivatives appear in \tilde{F}_{12} , since everything is charged adjointly with respect to the center of mass $U(1)$ gauge field. However, it is rather unusual that the gauge field B_i contributes to F_{12} , the field strength of the center of mass degree of freedom. The cause of this is the asymmetric fashion in which we are dealing with the brane and the anti-brane.

In a noncommutative field theory it is often convenient to work with the creation and annihilation coordinates

$$a = \frac{1}{\sqrt{2\theta}}(x^1 + ix^2), \quad a^+ = \frac{1}{\sqrt{2\theta}}(x^1 - ix^2) \quad (3.15)$$

satisfying $[a, a^+] = 1$. These are the analogue of complex coordinates $z = x^1 + ix^2$, $\bar{z} = x^1 - ix^2$. The gauge fields we introduced can also be written in the complex form

$$A_z = A_1 - iA_2, \quad A_{\bar{z}} = A_1 + iA_2. \quad (3.16)$$

It is straightforward to see that A_z always appears together with a^+ which can be regarded as ∂_z , and $A_{\bar{z}}$ always appears with a which can be written as $\partial_{\bar{z}}$.

4. The Tachyon

We expect the tachyon to arise as the off-diagonal entries of matrices X^1, X^2 . Modes from other matrices remain massless. In the brane-anti-brane system, the tachyon field is a complex scalar. However, one naively expects that any field arising from X^i carries a spacetime index tangent to the branes. The resolution of this puzzle is connected to the fact that the background (2.1) breaks the usual spacetime symmetry in the matrix model, and the conventional world-volume spacetime symmetry, as explained in the previous two sections, is a combination of the matrix model spacetime symmetry and gauge symmetry.

We work again with the harmonic representation. The gauge parameter associated with rotation generator can be written as

$$g = \begin{pmatrix} 0 & 0 \\ 0 & -\frac{\epsilon}{\theta}(2N+1) \end{pmatrix}, \quad (4.1)$$

where $N = a^+a$ is the number operator, ϵ is the infinitesimal rotation angle. As explained before, the rotation around point $z = x^1 + ix^2 = 0$ is generated by

$$\delta X^i = -\epsilon \epsilon^{ij} X^j + i[g, X^i]. \quad (4.2)$$

It is convenient to work with the complex matrices

$$Z = \frac{1}{\sqrt{2}}(X^1 + iX^2), \quad \bar{Z} = \frac{1}{\sqrt{2}}(X^1 - iX^2). \quad (4.3)$$

Since \bar{Z} is the Hermitian conjugate of Z , it is enough to work with Z . In the absence of the gauge fields,

$$Z = \sqrt{\theta} \begin{pmatrix} a & \tilde{T} \\ T & a^+ \end{pmatrix}. \quad (4.4)$$

Under the rotational transformation (4.2), Z transforms as

$$\delta Z = i\epsilon Z - i\epsilon\sqrt{\theta} \begin{pmatrix} 0 & -\tilde{T}(2N+1) \\ (2N+1)T & 2a^+ \end{pmatrix}. \quad (4.5)$$

We read from above that $\delta a = i\epsilon a$ and $\delta a^+ = -i\epsilon a^+$, meaning that a transforms as z and a^+ as \bar{z} . The off-diagonal fields transform according to

$$\delta T = -2i\epsilon NT, \quad \delta \tilde{T} = 2i\epsilon \tilde{T}(N+1). \quad (4.6)$$

These formulas say that under the rotation, both T and \tilde{T} transform quite nontrivially, and that one can not simply classify them by a tensor of a fixed rank.

T and \tilde{T} are both operator-valued, so it is convenient to expand them in terms of the basis $|m\rangle\langle n|$ where $N|m\rangle = m|m\rangle$.

$$T = \sum T_{mn}|m\rangle\langle n|, \quad \tilde{T} = \sum \tilde{T}_{mn}|m\rangle\langle n|. \quad (4.7)$$

Thus under a rotation, T_{mn} transforms as \bar{z}^{2m} and \tilde{T}_{mn} as $z^{2(n+1)}$. Since $m, n \geq 0$, one can never get a scalar from \tilde{T} , while T_{0n} are invariant under the rotation.

For a fixed m , one can write

$$\sum_n T_{mn}\langle n| = \langle 0|T_m \quad (4.8)$$

with an operator T_m , so that

$$T = \sum |m\rangle\langle 0|T_m. \quad (4.9)$$

Now T_m transforms as \bar{z}^{2m} . In particular, T_0 is a scalar under the rotation. Similarly,

$$\tilde{T} = \sum \tilde{T}_m|0\rangle\langle m|, \quad (4.10)$$

and \tilde{T}_m transforms as $z^{2(m+1)}$.

Although T_0 is invariant under rotation about $z = 0$, it is not translationally invariant. This is not a surprise, as T_0 itself is not a fully-fledged two dimensional field. The state $|0\rangle$ can be regarded as localized at $z = 0$. One can construct a similar state localized at another point z_0 satisfying

$$a|z_0\rangle = \frac{1}{\sqrt{2\theta}}z_0|z_0\rangle. \quad (4.11)$$

This state can be expressed as a coherent state

$$|z_0\rangle = e^{\frac{z_0}{\sqrt{2\theta}}a^+}|0\rangle. \quad (4.12)$$

In a sense, a^+ can be regarded as the generator of a translation in the z direction in the noncommutative plane. This observation agrees well with the gauge parameter (2.6), since $x^1 \sim a + a^+$. Just like $|0\rangle$, $|z_0\rangle$ is not well-localized in the \bar{z} direction, as the plane is noncommutative. Now a tachyon mode

$$T = |z_0\rangle\langle 0|T \quad (4.13)$$

is invariant under the rotation about the point z_0 . The corresponding gauge parameter similar to (4.1) can be expressed in terms new creation and annihilation operators $a \rightarrow a - z_0/\sqrt{2\theta}$, $a^+ \rightarrow a^+ - \bar{z}_0/\sqrt{2\theta}$.

The action (of strings) or the Hamiltonian (of D2-branes) is proportional to $[Z, \bar{Z}]^2$:

$$\begin{aligned} \frac{1}{\theta^2}[Z, \bar{Z}]^2 &= (1 - T^+T + \tilde{T}\tilde{T}^+)^2 + (1 - TT^+ + \tilde{T}^+\tilde{T})^2 \\ &\quad + 2|Ta^+ - aT + a^+\tilde{T}^+ - \tilde{T}^+a|^2, \end{aligned} \quad (4.14)$$

where the first two terms on the R.H.S. tell us that T is the tachyon while \tilde{T} is massive. That T is tachyonic was first noticed in [13]. The massive mode \tilde{T} should not be a propagating mode in the brane-anti-brane system. We now show that indeed this mode can be gauged away.

Although in the brane-anti-brane system, the gauge symmetry is $U(1) \times U(1)$, the full gauge symmetry in the matrix model is still $U(2)$, and T and \tilde{T} can be regarded as the corresponding off-diagonal gauge fields. Since the off-diagonal gauge parameter is a complex field, one can gauge away one of them. One would guess that one can gauge away any of them, or any linear combination of them, as in a brane-brane system. This is not the case here. We can gauge away only \tilde{T} , the massive mode. To see this, let us apply the gauge transformation with the gauge parameter

$$g = \begin{pmatrix} 0 & f^+ \\ f & 0 \end{pmatrix} \quad (4.15)$$

to Z , and we find

$$\begin{aligned} \delta T &= i(fa - a^+f) \\ \delta \tilde{T} &= i(f^+a^+ - af^+). \end{aligned} \quad (4.16)$$

Similar to (4.9) and (4.10), we expand

$$\begin{aligned} f &= \sum |m\rangle \langle 0| f_m \\ f^+ &= \sum f_m^+ |0\rangle \langle m|, \end{aligned} \quad (4.17)$$

where f_m, f_m^+ are operators. Using

$$a|m\rangle = \sqrt{m}|m-1\rangle, \quad a^+|m\rangle = \sqrt{m+1}|m+1\rangle$$

we find, in terms of the components

$$\begin{aligned} \delta \tilde{T}_m |0\rangle &= i(\sqrt{m+1}f_{m+1}^+ - af_m^+) |0\rangle \\ \langle 0| \delta T_m &= i\langle 0|(f_m a - \sqrt{m}f_{m-1}). \end{aligned} \quad (4.18)$$

For an arbitrary $\delta\tilde{T}_m$, it is possible to satisfy the first equation of (4.18), since one simply needs the following recursion relation

$$\sqrt{m+1}f_{m+1}^+ = -i\delta\tilde{T}_m + af_m^+. \quad (4.19)$$

Thus it is possible to gauge away any infinitesimal \tilde{T}_m by letting $\delta\tilde{T}_m = -\tilde{T}_m$. On the other hand, for an arbitrary δT_m , it is generally impossible to solve the second equation in (4.18). The reason is that in the recursion relation

$$\langle 0|f_m a = \langle 0|(-i\delta T_m + \sqrt{m}f_{m-1}) \quad (4.20)$$

a is not invertible. For instance, for a nonvanishing c-number valued δT_0 , there is no solution to the equation

$$\langle 0|f_0 a = -i\delta T_0 \langle 0|. \quad (4.21)$$

Now we choose a gauge in which $\tilde{T} = 0$. The residual gauge symmetry is $U(1) \times U(1)$. This is the desired gauge symmetry in the brane-anti-brane system. The action for T becomes quite simple in this gauge

$$\frac{1}{\theta^2}[Z, \bar{Z}]^2 = (1 - T^+T)^2 + (1 - TT^+)^2 + 2|Ta^+ - aT|^2. \quad (4.22)$$

Again, this action is not invariant under the world-volume rotation if we simply regard T as a scalar. As we showed before, T actually transforms nontrivially under a rotation. The breaking-down of an explicit rotational symmetry when T is treated as a scalar is due to the fact that in the matrix model the background forces us to use the star product which is natural only for the D-brane, not for the anti-D-brane. If an algebraic structure symmetric for the pair of brane and anti-brane exists, the tachyon should appear as a scalar, and should be related to the tachyon discussed here by a field redefinition, quite similar to the Seiberg-Witten field redefinition.

5. Tachyon Condensation

As discussed in the previous section, the complex tachyon field does not appear as a scalar in the matrix model solution. Naturally therefore, we do not expect a uniform tachyon condensation in this context, as also pointed out in [14]. First of all, we want to show that there does not exist a tachyon configuration making $[Z, \bar{Z}] = 0$, the absolute minimum, when gauge fields are set to zero.

From (4.22) the conditions for $[Z, \bar{Z}] = 0$ are

$$T^+T = 1, \quad a^+ = T^{-1}aT. \quad (5.1)$$

The first equation says that T is a unitary operator, and the second equations says that a^+ is similar to a . Taking Hermitian conjugate of this condition we have $a = T^{-1}a^+T$ or $a^+ = TaT^{-1}$. This together with the second equation in (5.1) implies

$$[T^2, a] = 0. \quad (5.2)$$

Similarly one can derive $[T^2, a^+] = 0$. Thus T^2 commutes with all operators constructed of a and a^+ and must be a c-number. Unitarity of T further implies that T^2 must be a pure phase: $T^2 = \exp(2i\phi)$. In general, $T = \text{diag}(\pm \exp(i\phi))$. But this form can not satisfy $a^+ = T^{-1}aT$. We conclude that there is no absolutely minimal tachyon condensation.

The cause of the absence of the minimal tachyon condensation is rather obvious, that is in the brane-anti-brane solution the gauge field strengths on the brane and the anti-brane have opposite sign. The lacking of a uniform minimal tachyon condensation is also noticed in [14]. Due to the nonscalar nature of T , we thus expect localized minimum at best, coming from solving the equation of motion derived from the variational principle $\delta \text{tr}[Z, \bar{Z}]^2 = 0$. The equation of motion is just

$$T(1 - T^+T) = NT + TN - aTa - a^+Ta^+ \quad (5.3)$$

and its Hermitian conjugate. It is not easy to find a general solution to the above equation. However, a simple solution making both sides of (5.3) vanish exists, it is simply

$$T = c|0\rangle\langle 0| \quad (5.4)$$

with $|c|^2 = 1$. This is a vortex localized at the origin. Unlike in the uniform situation studied in [3], $T \sim |m\rangle\langle m|$ can not be an exact solution, since both aTa and a^+Ta^+ bring T out of the projection operator. According to the discussion in the previous section, this is not surprising, since these modes are not rotationally invariant at all. Nevertheless, $T \sim |m\rangle\langle m|$ can be regarded as an approximate solution, since both aTa and a^+Ta^+ are slightly off-diagonal and quite close to NT and TN , and the R.H.S. of (5.3) almost cancel.

The energy difference between the brane-anti-brane configuration and a vortex on this system is given by

$$\Delta \text{tr}[Z, \bar{Z}]^2 = -2\theta^2 \text{tr}|0\rangle\langle 0| = -2\theta^2, \quad (5.5)$$

thus indeed the vortex solution is a lower energy configuration. On the other hand, the approximate solution $T = |m\rangle\langle m|$ carries more energy

$$\Delta \text{tr}[Z, \bar{Z}]^2 = 2(m-1)\theta^2. \quad (5.6)$$

So the system will not tend to create these nonisotropic vortices.

More vortex solutions can be simply generated by translating $|0\rangle\langle 0|$ to other points, using the matrix translation together with a gauge transformation discussed in sect.1.

It would be interesting to consider the situation when both gauge fields and the tachyon are turned on, and to look for nontrivial solutions to $[Z, \bar{Z}] = 0$. This will help to understand the enigmatic nature of the nothing state [15,16]. Since the tachyon can never be gauged away, as shown in the previous section, a nothing state involving nonvanishing tachyon can not be equivalent to a pure gauge field configuration. It is also interesting to see how the $U(1)$ confinement problem is resolved in the present context [17,18].

In conclusion, we have found that the brane-anti-brane system in matrix models has a rather strange representation. This strangeness is due to the asymmetric treatment of the brane and the anti-brane, and the noncommutative field theory is defined in favor of the brane. It is highly desirable to find a new algebraic structure incorporating different noncommutative structures on both branes. It may help to follow the line of [19] to use a similarity transformation to figure out this structure.

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